LTH 477
hep-ph/0003081

Fayet-Iliopoulos D-terms and anomaly mediated supersymmetry breaking

I. Jack and D.R.T. Jones

Dept. of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, U.K.

We show that in a minimal extension of the MSSM by means of an extra U_1 gauge group, the negative mass-squared problem characteristic of the Anomaly Mediated Supersymmetry Breaking scenario is naturally solved by means of Fayet-Iliopoulos D-terms. We derive a set of sum rules for the sparticle masses which are consequences of the resulting framework.

March 2000

The MSSM has (according to a recent census[1]) 124 parameters; an obvious embarrassment, and any (principled) reduction of this alarming total is obviously worthy of examination. Hence there has been interest in a specific and predictive framework wherein the gaugino masses M_a, the ϕ^3 coupling h^{ijk} and the $\phi\phi^*$-mass $m^i{}_j$ are all given in terms of a single mass parameter, m_0, and the β-functions of the unbroken theory by simple relations that are renormalisation group (RG) invariant. These results for the soft terms were (with the exception of the solution for the gaugino mass) first developed by seeking solutions to the exact β-function equations [2][3]; remarkably, it was then shown [4] [5] that they arise naturally if the supersymmetry-breaking terms originate in a vacuum expectation value for an auxiliary field in the supergravity multiplet. In this scenario, termed 'Anomaly Mediated Supersymmetry Breaking' (AMSB), m_0 is in fact the gravitino mass, and all the gaugino masses, soft $\phi\phi^*$ masses and A-parameters are determined in terms of it [6]–[14]. Unfortunately, however, a minimal implementation leads inevitably to negative (mass)^2 sleptons. The simplest resolution is the introduction of a common scalar (mass)^2, presumed to result from some other source of supersymmetry breaking. The advantage of this is that only one new parameter is introduced: the disadvantage is that RG-invariance of the soft mass prediction is sacrificed.

Here we propose an alternative solution in which the extra source of supersymmetry breaking arises spontaneously within the low energy effective field theory, by exploiting the fact that supersymmetric theories including U_1 factors have (in general) Fayet-Iliopoulos (FI) D-terms. In the MSSM, there is a non-zero FI-term, but this cannot solve the slepton problem because its (mass)^2 contributions to the LH and RH sleptons have opposite signs, being dictated by the hypercharge of the relevant field. Our proposed solution involves extending the MSSM to incorporate an extra U_1. It then becomes possible for both LH and RH sleptons to achieve the nirvana of positive (mass)^2 via FI contributions [1].

Theories with an extra U_1 have been studied as a means of parameterising deviations from the SM, and also for more positive reasons[2]. For example, in the supersymmetric case an extra U_1 can be used to explain the absence of dimension-4 R-parity violation (operators violating baryon and lepton number) [16]–[19]. Here we consider a minimal anomaly-free generalisation of the MSSM to the group $\mathcal{G}\otimes U_1'$, where $\mathcal{G} = SU_3\otimes SU_2\otimes U_1$,

[1] Use of FI terms is also a feature of Ref. [9], but in a different manner to that proposed here.

[2] We note the suggestion[15] that there are already "hints" of the existence of an extra Z' at around 1TeV.

with the addition of an unspecified number N of $\mathcal{G}$ singlets (S_i) and a superpotential W of the full theory given by

$$W = W_{MSSM} + W_S(S_i). \tag{1}$$

Here

$$W_{MSSM} = \mu_s H_1 H_2 + \lambda_t H_2 Q t^c + \lambda_b H_1 Q b^c + \lambda_\tau H_1 L \tau^c. \tag{2}$$

We retain Yukawa couplings only for the third generation, Q, L, t^c, b^c, τ^c, and we will denote the corresponding fields of the other generations by $\overline{Q}, E, u^c, d^c, e^c$. Let us define the U_1' hypercharges of the MSSM fields $Q, L, t^c, b^c, \tau^c, H_1, H_2$ to be $Y_Q', Y_L', Y_{t^c}', Y_{b^c}', Y_{\tau^c}', Y_{H_1}', Y_{H_2}'$. We will assume that the quark and lepton assignments are generation independent, i.e. $Y_L' = Y_E'$ etc; this means that our model will, in fact, naturally suppress dangerous flavour violating processes. It is straightforward to show that gauge invariance and absence of mixed gauge anomalies involving U_1' leads to the relations[18]:

$$\begin{aligned} Y_{H_1}' + Y_L' + Y_{\tau^c}' = Y_{H_2}' - Y_L' - Y_{\tau^c}' = 3Y_Q' + Y_L' = 0, \\ 3Y_{t^c}' + 2Y_L' + 3Y_{\tau^c}' = 3Y_{b^c}' - 4Y_L' - 3Y_{\tau^c}' = 0. \end{aligned} \tag{3}$$

(To obtain these relations it is not necessary to assume that $Y_{H_1}' + Y_{H_2}' = 0$: that is, gauge invariance of the μ_s-term is a consequence of the framework[18].) To cancel the $(U_1')^3$ and U_1'-gravitational anomalies, the U_1' hypercharges s_i of the fields S_i must satisfy the constraints:

$$\sum_{i=1}^{N} s_i = -3(2Y_L' + Y_{\tau^c}'), \quad \text{and} \quad \sum_{i=1}^{N} s_i^3 = -3(2Y_L' + Y_{\tau^c}')^3. \tag{4}$$

Suppose we prefer hypercharges to be rational; then the classification of solutions to Eq. (4) is an example of a well-known problem: finding the rational points on a n-dimensional surface. For example the rational points on the circle $x^2 + y^2 = 1$ are given by

$$(x, y) = (0, -1) \quad \text{and} \quad \left(\frac{2q}{1+q^2}, \frac{1-q^2}{1+q^2}\right) \tag{5}$$

where q is rational. The case $N = 3$ of Eq. (4) was analysed in Ref. [18]; the solution is

$$\begin{aligned} (s_1, s_2, s_3) &= -(2Y_L' + Y_{\tau^c}')(1, 1, 1) \quad \text{and} \\ (s_1, s_2, s_3) &= -(2Y_L' + Y_{\tau^c}')\left(\frac{5+3q^2}{q^2-1}, \frac{q^2+q+4}{q+1}, -\frac{q^2-q+4}{q-1}\right) \end{aligned} \tag{6}$$

where again q is rational. We will simply assume that for some N there exists an appropriate solution, and that the singlet sector provides the Z' vector boson with a sufficiently large mass term so that its mixing with the Z is adequately suppressed.

For simplicity we also choose to impose the condition $\mathrm{Tr}(YY') = 0$. This prevents mixing of the U_1 and U_1' kinetic terms for the gauge bosons (through the one loop approximation)[3] and leads to the relation:

$$3Y_L' + 7Y_{\tau^c}' = 0 \tag{7}$$

The resulting hypercharges are shown in Table 1, with the U_1 ones for comparison:

	Q	L	t^c	b^c	τ^c	H_1	H_2	S_i
Y	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$\frac{1}{3}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0
Y'	$\frac{7}{3}$	-7	$\frac{5}{3}$	$-\frac{19}{3}$	3	4	-4	s_i

Table 1: The U_1 and $U_1{}'$ hypercharges.

With this assignment we indeed prevent the dimension-4 R-parity violating operators.

In a theory with anomaly-generated soft parameters, and with FI terms $\xi_1 D$, $\xi_2 D'$ for U_1 and U_1' respectively, a soft mass for a generic field is given after elimination of the D-terms by $m^2 + g'\xi_1 Y + g''\xi_2 Y'$, where

$$m^2 = \tfrac{1}{2}|m_0|^2 \mu \frac{d}{d\mu}\gamma, \tag{8}$$

with γ being the anomalous dimension. (We denote the gauge couplings for $SU(3)$, $SU(2)$, the MSSM U_1 and the new U_1' by g_3, g_2, $g_1 = \sqrt{\frac{5}{3}}g'$, and g'' respectively.) Consequently, after spontaneous symmetry breaking, the effective soft masses of the squarks and sleptons (before including A-parameter and μ_s-term mixing effects) are given by

$$\begin{aligned}
\overline{m}_Q^2 &= m_Q^2 + \tfrac{1}{6}\zeta_1 + \zeta_2 Y_Q', \quad & \overline{m}_{t^c}^2 &= m_{t^c}^2 - \tfrac{2}{3}\zeta_1 + \zeta_2 Y_{t^c}', \\
\overline{m}_{b^c}^2 &= m_{b^c}^2 + \tfrac{1}{3}\zeta_1 + \zeta_2 Y_{b^c}', \quad & \overline{m}_L^2 &= m_L^2 - \tfrac{1}{2}\zeta_1 + \zeta_2 Y_L', \\
\overline{m}_{\tau^c}^2 &= m_{\tau^c}^2 + \zeta_1 + \zeta_2 Y_{\tau^c}',
\end{aligned} \tag{9}$$

[3] The consequences of this kinetic mixing have been studied in Ref [20].

where

$$\zeta_1 = g'[\xi_1 - g'\tfrac{1}{4}(v_1^2 - v_2^2)], \quad \zeta_2 = g''[\xi_2 + \mathcal{S} + 2g''(v_1^2 - v_2^2)], \tag{10}$$

and where

$$m_Q^2 = \tfrac{1}{2}|m_0|^2\mu\frac{d}{d\mu}\gamma_Q, \quad m_{t^c}^2 = \tfrac{1}{2}|m_0|^2\mu\frac{d}{d\mu}\gamma_{t^c}, \tag{11}$$

and so on. It is easy to write down the analogous expressions for the other generations. We have included in Eqs. (9),(10) the standard D-term contributions to the masses resulting from the Higgs vevs, together with a contribution $\mathcal{S}$ from the (unknown) vevs of the singlets S_i. Note that the dependence on the singlet sector is subsumed into ζ_2, and therefore much of the discussion can be independent of the precise structure of the singlet terms.

The relation between each $\overline{m}^2$ and m^2 in Eq. (9) is quite generally RG invariant (it is important that the $\beta_{\overline{m}^2}, \beta_{m^2}$ are calculated with D eliminated and D uneliminated respectively [21]). It is also invariant if we replace $\zeta_{1,2}$ by constants (but in this case both $\beta_{\overline{m}^2}$ and β_{m^2} are calculated with D uneliminated). Thus in a general theory with $\mathcal{N}$ non-anomalous U_1 factors, then the relation

$$(\hat{m}^2)^i{}_j = (m^2)^i{}_j + m_0^2\delta^i{}_j \tag{12}$$

is not RG invariant (for constant m_0^2), but

$$(\hat{m}^2)^i{}_j = (m^2)^i{}_j + m_0^2\sum_{a=1}^{\mathcal{N}} k_a(Y_a)^i{}_j \tag{13}$$

is RG invariant. This is easily shown using the gauge invariance and anomaly cancellation conditions, together with the general formula for β_{m^2} given, for example in Ref. [21]. Evidently this invariance continues to hold in the limit that the U_1 gauge couplings approach zero, so we do not even need the U_1 groups to be gauged (or to impose relations like Eq. (7), so that we could then have the same sign for Y'_L and Y'_{τ^c}); though clearly it would be artificial to impose anomaly cancellation conditions in that case. String theories often give rise to apparently *anomalous* U'_1 symmetries, with the anomaly cancelled by the Green-Schwarz mechanism. We might therefore entertain the possibility of dispensing with the singlet sector and invoking the GS mechanism to cancel the $(U'_1)^3$ and U'_1-gravitational anomalies (see Eq. (4)). If the U'_1 symmetry were broken at a high mass scale, the only low-energy residue of the U'_1 would be the FI terms. However, we would then lack a rationale for imposing cancellation of the mixed gauge anomalies, a cancellation necessary to make Eq. (13) RG invariant. We will therefore persist with a gauged, non-anomalous U'_1.

The gaugino mass for a gauge coupling g (either g_3, g_2, g_1 or g'') in the AMSB scenario is given by [4]

$$M_g = m_0 \frac{\beta_g}{g}. \tag{14}$$

Moreover, the A-parameters are given by

$$A_t = -m_0(\gamma_Q+\gamma_{t^c}+\gamma_{H_2}), \quad A_b = -m_0(\gamma_Q+\gamma_{b^c}+\gamma_{H_1}), \quad A_\tau = -m_0(\gamma_L+\gamma_{\tau^c}+\gamma_{H_1}). \tag{15}$$

(We could write down similar results for the first two generation A parameters, but they will have no impact on our calculations since the corresponding Yukawa couplings are small.)

For completeness we record here the expressions for the anomalous dimensions:

$$\begin{aligned}
16\pi^2\gamma_{H_1} &= 3\lambda_b^2 + \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2 - 2Y_{H_1}'^2 g''^2, \\
16\pi^2\gamma_{H_2} &= 3\lambda_t^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2 - 2Y_{H_2}'^2 g''^2, \\
16\pi^2\gamma_{L} &= \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2 - 2Y_{L}'^2 g''^2, \\
16\pi^2\gamma_{Q} &= \lambda_b^2 + \lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{3}{2}g_2^2 - \tfrac{1}{30}g_1^2 - 2Y_{Q}'^2 g''^2, \\
16\pi^2\gamma_{t^c} &= 2\lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{8}{15}g_1^2 - 2Y_{t^c}'^2 g''^2, \\
16\pi^2\gamma_{b^c} &= 2\lambda_b^2 - \tfrac{8}{3}g_3^2 - \tfrac{2}{15}g_1^2 - 2Y_{b^c}'^2 g''^2, \\
16\pi^2\gamma_{\tau^c} &= 2\lambda_\tau^2 - \tfrac{6}{5}g_1^2 - 2Y_{\tau^c}'^2 g''^2.
\end{aligned} \tag{16}$$

In the tree approximation the μ_s-term is given by the Higgs minimisation condition:

$$\mu_s^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}M_W^2 + \sec 2\beta \left(\frac{1}{2}\zeta_1 - 4\zeta_2\right). \tag{17}$$

The masses of the pseudoscalar and charged Higgs bosons are given at leading order by the usual expressions

$$m_A^2 = 2r_1, \quad m_{H^\pm}^2 = 2r_1 + M_W^2 \tag{18}$$

where we define

$$r_1 = \tfrac{1}{2}(m_{H_2}^2 + m_{H_1}^2) + \mu_s^2. \tag{19}$$

The other minimisation condition,

$$m_3^2 = r_1 \sin 2\beta \tag{20}$$

[4] The significance of the *sign* of the gluino mass term is investigated in Ref. [11].

determines the soft H_1H_2 mass term. In fact there exists an RG invariant solution for this as well[2]:

$$m_3^2 = -m_0\mu\frac{d}{d\mu}\mu_s. \tag{21}$$

We find, however, that there is no value of m_0 leading to an otherwise acceptable spectrum and a result for $\tan\beta$ satisfying Eq. (20). Thus, in common with previous work on the AMSB scenario, we are obliged to assume that m_3^2 arises from an alternative source of supersymmetry breaking, presumably linked to the μ_s-term. It is also possible to construct (perturbatively) a RG trajectory for $\xi_{1,2}$ so that $\xi_{1,2} \sim m_0^2$ [21], but the resulting values of $\zeta_{1,2}$ are too small for our purpose here.

We choose to normalise the U_1' hypercharge so as to satisfy at the weak scale the relation

$$\mathrm{Tr}(Y^2 g_1^2) = \mathrm{Tr}(Y'^2 g''^2), \tag{22}$$

which corresponds to equal U_1 and U_1' gaugino masses. We will present results for the case when the $\sum s_i^2$ is large, so that the U_1' gauge coupling is small; this limit suppresses $Z-Z'$ mixing, while allowing a large Z' mass (because $\sum s_i^2$ is large); though of course in this limit the Z' would decouple in any case.

Let us now consider the nature of the predicted mass spectrum. The heaviest sparticle masses scale with m_0 and are given roughly by $M_{\mathrm{SUSY}} = \frac{1}{40}m_0$. Consequently we take account of leading-log corrections by evaluating the mass spectrum at the scale M_{SUSY}. In other words, before applying Eqs. (9), (17) etc., we evolve the dimensionless couplings (together with v_1, v_2) from the weak scale up to the scale M_{SUSY}. In order that the sleptons have positive $(\text{mass})^2$, we require

$$m_E^2 - \tfrac{1}{2}\zeta_1 + \zeta_2 Y_L' > 0, \quad \text{and} \quad m_{e^c}^2 + \zeta_1 + \zeta_2 Y_{\tau^c}' > 0, \tag{23}$$

where m_E^2 and $m_{e^c}^2$ are the standard AMSB expressions as in Eq. (11). It turns out that the most important other constraint comes from requiring $m_A^2 > 0$. This constraint, together with Eq. (23), define a triangular region in the ζ_1, ζ_2 plane. For $m_0 = 40\mathrm{TeV}$, and for $\tan\beta = 5$, this triangular region is shown in Fig. 1.

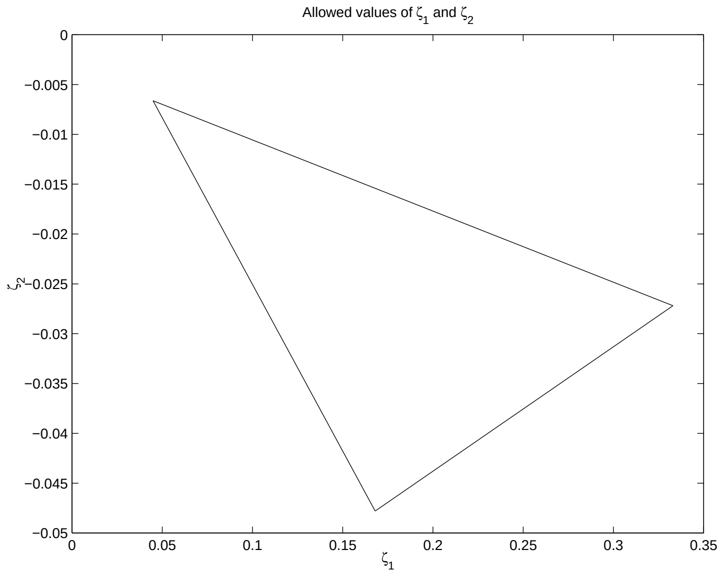

Fig. 1: Allowed values of ζ_1 and ζ_2, for $\tan\beta = 5$ and $m_0 = 40$TeV.

For a choice of $\zeta_1 = 0.2$, $\zeta_2 = -.02$, we find $|\mu_s| = 645$GeV and (choosing $\mu_s > 0$) a mass spectrum given by:

$$\begin{gathered} m_{\tilde{t}_1} = 575, \quad m_{\tilde{t}_2} = 861, \quad m_{\tilde{b}_1} = 825, \quad m_{\tilde{b}_2} = 1040, \quad m_{\tilde{\tau}_1} = 137, \quad m_{\tilde{\tau}_2} = 339, \\ m_{\tilde{u}_L} = 931, \quad m_{\tilde{u}_R} = 851, \quad m_{\tilde{d}_L} = 935, \quad m_{\tilde{d}_R} = 1045, \quad m_{\tilde{e}_L} = 139, \quad m_{\tilde{e}_R} = 339, \\ m_{\tilde{\nu}} = 112, \quad m_A = 453, \quad m_{H^\pm} = 461, \quad m_{\tilde{\chi}^\pm_{1,2}} = 104, 649 \quad m_{\tilde{g}} = 1007, \end{gathered} \tag{24}$$

where all masses are given in GeV. The sleptons $\tilde{\tau}_1$ and $\tilde{e}_L$ are light because we have chosen a point relatively near one edge. Alternative choices of $\zeta_{1,2}$ in the interior of the allowed triangle lead to a generally similar spectrum; well away from the edges $m_{\tilde{\tau}_1}$ and $m_{\tilde{e}_L}$ approach 300GeV. The CP-even Higgs and neutralino masses are sensitive to the singlet sector so we cannot specify them precisely. However based on the arguments of, for example, Ref. [22] there will be an upper bound on the lighter Higgs of around 140GeV. Because M_2 is the smallest gaugino mass, we also expect a light neutralino approximately degenerate with the light chargino (both being predominantly wino in content) at around 104GeV, with the chargino being heavier due to radiative corrections. The light neutralino may be the LSP; the resulting distinctive phenomenology and the characteristic decay $\tilde{\chi}^\pm \to \tilde{\chi}^0 + \pi^\pm$ are described in Refs. [7], [14], [23], [24].

In a limit such that the singlet sector decouples, the CP even and neutralino spectrum become calculable and we obtain (for the same values of $\zeta_{1,2}$)

$$m_{h,H} = 88, 455\text{GeV} \tag{25}$$

and

$$m_{\tilde{\chi}_{1\cdots4}} = 103, 366, 648, 658\text{GeV}. \tag{26}$$

As usual a complete calculation of the radiative corrections to m_h may be expected to result in a somewhat higher value. In this scenario there is no motivation for imposing Eq. (7); but choosing a different set of Y' satisfying Eq. (3) simply amounts to a different choice of co-ordinates for the (ζ_1, ζ_2) plane.

If $\zeta_{1,2}$ were to correspond to a point near one of the two appropriate edges of the triangle, the LSP would be a charged scalar lepton. Of course anomalous heavy isotope searches suggest that that a charged LSP is unlikely, but for a contrarian viewpoint on this issue, see for example Ref. [25], which is also of interest in that it considers the phenomenological footprints of a FI term in the MSSM.

As previous authors have observed[7], m_E^2 and $m_{e^c}^2$ are very nearly equal; this does not extend to the physical masses $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$ in our framework, because of the FI contributions (the same observation applies to some other resolutions of the tachyonic slepton problem, see e.g. Ref. [6]). Finally, the lightest strongly-interacting particle is the lighter stop, $\tilde{t}_1$; but this is a feature of much of MSSM parameter space.

As we reduce m_0, or increase $\tan\beta$, the triangular region of $\zeta_{1,2}$ satisfying Eq. (23) and $m_A^2 > 0$ diminishes, and moreover, experimental constraints on $m_{\tilde{\chi}_1^\pm}$ or $m_{\tilde{\tau}_1}$ further reduce the allowed region for smaller m_0 or large $\tan\beta$ respectively. In fact, we find that an acceptable spectrum is only possible for $m_0 \geq 35\text{TeV}$ (with $\tan\beta = 5$) or for $\tan\beta \leq 27$ (with $m_0 = 40\text{TeV}$). For smaller $\tan\beta$, the spectrum is similar to Eq. (24), but the allowed triangle begins to shrink as $\tan\beta \to 2$, a value approaching (as it happens) the quasi-infra-red fixed point for λ_t.

We have taken g'' very small by taking $\sum s_i^2$ large and imposing Eq. (22). For larger values of g'' the allowed parameter space is still determined by the triangle, and the broad features of the spectrum remain the same.

The most distinctive feature of the model presented here is the existence of sum rules for combinations of masses in which the dependence on $\zeta_{1,2}$ cancels. We find

$$\begin{aligned} \overline{m}_L^2 + 3\overline{m}_Q^2 &= m_L^2 + 3m_Q^2, \\ \overline{m}_{t^c}^2 + \overline{m}_{b^c}^2 + 2\overline{m}_Q^2 &= m_{t^c}^2 + m_{b^c}^2 + 2m_Q^2, \\ \overline{m}_{t^c}^2 + \overline{m}_{\tau^c}^2 - 2\overline{m}_Q^2 &= m_{t^c}^2 + m_{\tau^c}^2 - 2m_Q^2, \end{aligned} \tag{27}$$

where $\overline{m}^2$ are the effective soft mass parameters and m^2 are the pure AMSB masses as given by Eq. (11). From these results we can obtain the following relations for the physical masses:

$$\begin{aligned} m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 + m_{\tilde{b}_1}^2 + m_{\tilde{b}_2}^2 - 2(m_t^2 + m_b^2) &= 2.79\left(\frac{m_0}{40}\right)^2 \text{TeV}^2 \\ m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_2}^2 + m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 - 2(m_t^2 + m_\tau^2) &= 1.15\left(\frac{m_0}{40}\right)^2 \text{TeV}^2. \end{aligned} \tag{28}$$

Similar results apply for the first two generations as follows:

$$\begin{aligned} m_{\tilde{e}_L}^2 + 2m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 &= m_E^2 + 3m_Q^2 = 2.63\left(\frac{m_0}{40}\right)^2 \text{TeV}^2, \\ m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 + m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 &= m_{u^c}^2 + m_{d^c}^2 + 2m_Q^2 = 3.56\left(\frac{m_0}{40}\right)^2 \text{TeV}^2, \\ m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 - m_{\tilde{u}_R}^2 - m_{\tilde{e}_R}^2 &= 2m_Q^2 - m_{u^c}^2 - m_{e^c}^2 = 0.90\left(\frac{m_0}{40}\right)^2 \text{TeV}^2. \end{aligned} \tag{29}$$

Note that in Eqs. (28), (29) the dependence of the physical masses on M_W^2 has cancelled in the combinations on the left-hand side, in addition to the dependence on $\zeta_{1,2}$. Finally, sum rules involving the CP-odd Higgs:

$$m_A^2 - 2\sec 2\beta\left(m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2\right) = \sec 2\beta\left[m_{H_2}^2 - m_{H_1}^2 - 2(m_{e^c}^2 + m_E^2)\right] = 0.49\left(\frac{m_0}{40}\right)^2 \text{TeV}^2, \tag{30}$$

and

$$\begin{aligned} m_A^2 - 2\sec 2\beta\left(m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_2}^2 - 2m_\tau^2\right) &= \sec 2\beta\left[m_{H_2}^2 - m_{H_1}^2 - 2(m_{\tau^c}^2 + m_L^2)\right] \\ &= 0.49\left(\frac{m_0}{40}\right)^2 \text{TeV}^2. \end{aligned} \tag{31}$$

(The numerical results above apply for $\tan\beta = 5$.) We have demonstrated that it is possible to construct a viable model by combining the AMSB scenario with FI D-terms in a model with an extra U_1. The model incorporates natural flavour conservation and suppression of proton decay. One might imagine a more elegant version of the model which forbade the μ_s-term, and incorporated neutrino masses; this is not possible, however, without introducing fields which are MSSM non-singlets[18]. A recent version of this idea (not in the AMSB context) is to be found in Ref. [26]; however because in this case SU_3 is not asymptotically free due to the presence of extra colour triplets, it is hard (in the AMSB framework) to achieve an acceptable vacuum.

Acknowledgements

This work was supported in part by the Leverhulme Trust. We thank Nigel Backhouse, Victor Flynn and Roz Wild for Diophantine discussions.

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